Quantum gloves: Physics and Information

N. Gisin

Group of Applied Physics, University of Geneva, 1211 Geneva 4, Switzerland

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The slogan *information is physical* has been so successful that it led to some excess. Classical and quantum information can be thought of independently of any physical implementation. Pure information tasks can be realized using such abstract c- and qu-bits, but physical tasks require appropriate physical realizations of c- or qu-bits. As illustration we consider the problem of communicating chirality.

Assume that two distance partners like to compare the chiralities of their Cartesian reference frames. This is impossible by exchanging only classical information, i.e. by sending only abstract 0's and 1's. This is true in the most general relativistic context [1]. But for our purpose it suffice to consider a Newtonian physics worldview. It is then quite intuitive to grasp why information about chirality can't be encoded in classical bits: bits measure the quantity of information, but have per se no meaning, in particular no meaning about geometric and physical concepts. Hence, if our world is invariant under *left* ↔ *right*, then mere information is unable to distinguish between *left* and *right*. Now, information is physical, as Landauer used to emphasize and as every physicists knows today, hence let's consider classical bits physically realized in some system. For example the bits 0 and 1 could be realized by right-handed and left-handed gloves, respectively. It is obvious that such physical bits can be used to send chirality information. But bits realized by black and white balls couldn't do the job.

Let us now consider what quantum information brings to the situation under investigation. Note that recently many authors did consider similar formal problems, like aligning reference frames, without paying too much attention to the deep physical meaning of the problem. At first sight one may quickly conclude that sending abstract superpositions of basic kets $|0\rangle$ and $|1\rangle$, i.e. qubits $c_0|0\rangle + c_1|1\rangle$ with complex amplitudes c_j, doesn't help for the same reason as classical bits: they have no geometric nor physical meaning. The quantum situation is however more tricky because of the phenomenon of entanglement: Alice and Bob (we adopt the by now traditional labelling of the two partners) may exchange many qubits in such a way as to share many entangled qubit pairs in the singlet state: $|0,1\rangle - |1,0\rangle$. For clarity assume first that Alice and Bob both use the same reference frame. Then, using local operations and classical communication (LOCC), they can measure all the correlations $\langle \sigma_j \otimes \mathbb{1}\rangle$, $\langle \mathbb{1} \otimes \sigma_k\rangle$ and $\langle \sigma_j \otimes \sigma_k\rangle$, $j,k = x,y,z$ where σ_x, σ_y and σ_z denote the 3 Pauli matrices. From these measured correlations Alice and Bob compute the matrix:

$$\frac{1}{4}\left(\mathbb{1} + \langle\vec{\sigma}\otimes\mathbb{1}\rangle\vec{\sigma}\otimes\mathbb{1} + \langle\mathbb{1}\otimes\vec{\sigma}\rangle\mathbb{1}\otimes\vec{\sigma} + \langle\sigma_j\otimes\sigma_k\rangle\sigma_j\otimes\sigma_k\right) \quad (1)$$

which is nothing but the density matrix representing the quantum state they share, the projector onto the singlet state in our example. Consequently, the 4 eigenvalues of (1) are positive (≥ 0). Next, if Bob's reference frame is rotated compared to Alice's one, but still of the same chirality, then the matrix (1) represents the singlet state with one part accordingly rotated by a unitary transformation; the 4 eigenvalues would still be positive. What now if Bob's axes are all opposite to Alice's ones: $\vec{r}_{Bob} = -\vec{r}_{Alice}$? This corresponds to Bob using a frame with opposite chirality with respect to that of Alice. In such a case it might seem that if Alice and Bob proceed exactly as above, they would end up with the following computed matrix:

$$\frac{1}{4}\left(\mathbb{1} + \langle\vec{\sigma}\otimes\mathbb{1}\rangle\vec{\sigma}\otimes\mathbb{1} - \langle\mathbb{1}\otimes\vec{\sigma}\rangle\mathbb{1}\otimes\vec{\sigma} - \langle\sigma_j\otimes\sigma_k\rangle\sigma_j\otimes\sigma_k\right) \quad (2)$$

(notice the 2 sign changes!). This amounts to change $\langle\vec{\sigma}\rangle$ to $-\langle\vec{\sigma}\rangle$ on Bob's side which is equivalent, up to a unitary transformation, to the well known partial transposition introduced by Peres [2] as a criteria for entanglement. And indeed the matrix (2) has a negative eigenvalue whenever the original state (1) is entangled [3]. This observation led Lajos Diosi to the initial conclusion that pure quantum information and entanglement allows one to compare chiralities. But, as Diosi himself noticed soon after posting [4], this is not quite so. Indeed, the Poincaré sphere that we did implicitly use above does not float in our "real" 3-dimensional space, but is a convenient mathematical construction. Abstract qubits don't point in any direction and thus can't be used to define spatial directions nor chirality.

Let us now assume that the qubits are physically realized by spins $\frac{1}{2}$. One may think that now we have spatial directions and can thus use entanglement to define chirality. But this is still not quite so [5, 6]. Indeed, consider a spin $\frac{1}{2}$ particle in a typical Stern-Gerlach experiment and assume it is deflected towards the ceiling. From this elementary fact one can't conclude that the resulting spin state is "up"; in fact the relevant direction is that of the gradient of the magnetic field. If a physicist using a right-handed reference frame correctly concludes from the above fact that the spin is "up", then a physicist using a left-handed frame would equally correctly conclude that the spin is "down", because the latter physicist "sees" a magnetic field pointing in the opposite direction than that assigned (by convention!) by the first physicist. This is not special to quantum physics, the same holds in classical physics for all axial vectors, like e.g. angular momentum. Hence, spins are not appropriate realizations of qubit if the task is to communicate chirality. Rather than an axial vector one should use a polar vector like the vector joining an electron in an atom to its nucleus. This can be done using the orbital angular momenta Y_m^l. But before elaborating on this, let us look for the closest quantum equivalent to the glove implementation of classical bits discussed in the introduction.

In order to proceed, we borrow from [7] the concept of *quantum gloves*, that is elementary quantum states that contain nothing but the abstract concept of chirality, let us introduce the following chirality operator [7]:

$$\chi = \frac{1}{2\sqrt{3}}(\sigma_x\otimes\sigma_y\otimes\sigma_z+\sigma_y\otimes\sigma_z\otimes\sigma_x+\sigma_z\otimes\sigma_x\otimes\sigma_y - \sigma_x\otimes\sigma_z\otimes\sigma_y-\sigma_z\otimes\sigma_y\otimes\sigma_x-\sigma_y\otimes\sigma_x\otimes\sigma_z) \quad (3)$$

Note that all positive permutations appear with positive signs, while all negative permutations have a negative sign. This chirality operator has 3 eigenvalues: 0 and ±1, hence $\chi=\chi_+-\chi_-$. If Alice sends the mixed state $\rho_+=\frac{1}{2}\chi_+$ corresponding to the projector onto the 2-dimensional eigenspace associated to the eigenvalue $+1$ and if Bob measures χ, then Bob obtains the results $+1$ if and only if he uses the same chirality as Alice [10]. But, if the qubits are realized with axial vectors, like spins, then knowing χ is equivalent to knowing the chirality, hence in such a case Bob can measure χ only if he does already know Alice's chirality! Accordingly the states $\rho_\pm$ are not good abstract *quantum gloves*, for this we need polar vectors, i.e. vectors that really point in some spatial direction. Since we are not interested in any radial variable, we now on use the spherical harmonics Y_m^l. Under reflection about the origin, $P\vec{n}=-\vec{n}$, they transform as $PY_m^l=(-1)^lY_m^l$. Consider the two following 4-particle (e.g. 3 electrons and one nucleus) rotationally invariant states [7]:

$$|S\rangle = Y_0^0Y_0^0Y_0^0 \quad (4)$$

$$|A\rangle = \frac{1}{\sqrt{6}}\big(Y_1^1Y_0^1Y_{-1}^1+Y_0^1Y_{-1}^1Y_1^1+Y_{-1}^1Y_1^1Y_0^1 -Y_1^1Y_{-1}^1Y_0^1-Y_{-1}^1Y_0^1Y_1^1-Y_0^1Y_1^1Y_{-1}^1\big) \quad (5)$$

Clearly $P|S\rangle=|S\rangle$ and $P|A\rangle=-|A\rangle$. The following two states can thus be defined as *quantum gloves*:

$$|G^\pm\rangle=\frac{|S\rangle\pm|A\rangle}{\sqrt{2}}, \quad (6)$$

they do indeed transform properly: $P|G^+\rangle=|G^-\rangle$ and $P|G^-\rangle=|G^+\rangle$!

Note that the two states $|S\rangle$ and $|A\rangle$ can be used to define qubits $c_0|S\rangle+c_1|A\rangle$: each qubit is realized with 4 particles. Space reflections act on such qubits as phase-flips: $P(c_0|S\rangle+c_1|A\rangle)=c_0|S\rangle-c_1|A\rangle$. The chirality operator χ is then invariant under spatial reflection: $P\otimes P\otimes P\chi P\otimes P\otimes P=\chi$. Hence, using such realized qubits, Bob could measure χ without knowing Alice's chirality. But this requires the use of 3x4=12 particles, the states (6) are thus much simpler.

Like classical information, quantum information per se is not physical. As emphasized by Landauer, information – including quantum information – requires a physical implementation. Depending on the task, some implementations are advantageous. The advantage can be so large as to render possible seemingly impossible tasks. In the present context, the determination of chirality is impossible using some implementations of classical bits or of quantum bits, like e.g. optical pulses and spin respectively, but is perfectly possible both using c-bits or qubits provided appropriate physical implementations are used, like e.g. gloves and superpositions of excited atomic states, respectively [11].

In conclusion, quantum information can be thought of independently of any implementation, similarly to classical information. This rather trivial remark implies that quantum information can only achieve tasks which are expressed in pure information theoretical terms, like cloning and factoring, but can't perform physical tasks like aligning reference frames [5, 6] or defining temperature. This stresses that quantum teleportation is an information concept and does not permit the teleportation of a physical object, including its mass and chirality. This underlines that *information is physical*, but *physics is more than mere information* [9].

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[1] Feynman's talk "Symmetry in physical laws" was published by R.P.Feynman,R.B.Leighton,M.Sands: The Feynman Lectures on Physics, vol.I. Chap.52 (Addison-Wesley, 1963) or, e.g., in R.P.Feynman: Six Not-So-Easy Pieces: Einstein's Relativity, Symmetry, and Space-Time (Perseus Books Group, 1998). In this note we ignore particle physics.
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[3] M. Horodecki, P. Horodecki and R. Horodecki, Phys. Lett. A **223**, 1 (1996).
[4] L. Diosi, quant-ph/0007046.
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[6] D. Collins and S. Popescu, quant-ph/0401096.

[7] D. Collins, L. Diosi, N. Gisin, S. Massar and S. Popescu, *Quantum gloves*, in preparation.

[8] P. Zanardi and M. Rasetti, Phys. Rev. Lett. **79**, 3306 (1997); D.A. Lidar, I.L. Chuang and K.B. Whaley, Phys. Rev. Lett. **81**, 2594 (1998).

[9] As far as I know, no one has been able to rigorously define *abstract quantum information*. It seems likely that the distinction between *information tasks* and *physical tasks* is equally hard to draw. But it certainly deserves to be investigated.

[10] Note that both states $\rho_\pm$ are invariant under global rotation; they could thus be used for *decoherence free* [8] quantum communication.

[11] Note that a physicist not polluted by excessive use of quantum information would probably come up with a much simpler solution: send a circularly polarized photon, the momentum provides the polar vector and the circular polarization the axial vector, a seemingly minimal set of 2 vectors.