Is quantum theory universally valid?

A. Peres and W. H. Zurek

Citation: **50**, 807 (1982); doi: 10.1119/1.13086
View online: http://dx.doi.org/10.1119/1.13086
View Table of Contents: http://aapt.scitation.org/toc/ajp/50/9
Published by the American Association of Physics Teachers

an atomic gas at the onset of Bose–Einstein condensation. In this case we can set $\lambda = 1$ and yet ignore the ground state. By using $q(n\beta) = n^{-3/2}q$ we then find, from Eqs. (6) and (8),

$$N_n/N = n[n^{-3/2} - (n+1)^{-3/2}]/\zeta(3/2), \tag{13}$$

with ζ the Riemann zeta function, $\zeta(3/2) \simeq 2.612$. Thus the fraction of atoms in orbitals of occupancy n depends only on n: $N_1/N \simeq 0.25$, $N_2/N \simeq 0.12$, $N_3/N \simeq 0.08$, etc.

The results (4)–(7) apply even to the region of Bose–Einstein condensation [characterized by $N/q > \zeta(3/2)$ for an atomic gas] in which the average number M of molecules in the ground state is macroscopically large. This may seem paradoxical since there is no sign of any special behavior of N_n around $n = M$. The explanation is that the fluctuation of the number of molecules in the (nondegenerate) ground state, $(M^2 + M)^{1/2}$, is itself macroscopically large. In fact, the probability of finding n molecules in the ground state decreases with n, but only very slowly: it is $(M+1)^{-1}$ for $n = 0$ and approximately $e^{-1}(M+1)^{-1}$ for $n = M$, for example. The contribution to N_n from the molecular ground state has a very flat maximum near $n = M$.

Outside the region of Bose–Einstein condensation the average values N_n and fluctuations δN_n form a conceptually interesting characterization of the quantum corrections to an ideal boson gas. The expressions (6) and (7) apply to open systems but we expect them to hold to a good approximation also for closed isothermal or isolated systems (with the appropriate definitions of μ and β) with the exception of the expression for δN_1.

[1] H. Kroemer, Am. J. Phys. **48**, 962 (1980).
[2] F. Hynne, Am. J. Phys. **49**, 125 (1981).
[3] A. G. McLellan, Am. J. Phys. **40**, 704 (1972).
[4] D. L. Ford, Am. J. Phys. **39**, 215 (1971).
[5] R. Becker, *Theory of Heat* (Springer, Berlin, 1967).
[6] P. G. Hoel, S. C. Port, and C. J. Stone, *Introduction to Probability Theory* (Houghton Mifflin, Boston, 1971), Sec. 4.4 and example 5, p. 52.

Is quantum theory universally valid?

A. Peres[a] and W. H. Zurek[b]
Center for Theoretical Physics, The University of Texas at Austin, Austin, Texas 78712

(Received 24 August 1981; accepted for publication 20 November 1981)

Quantum theory has been criticized for not being deterministic and therefore not universal: it cannot completely describe a measurement aimed at verifying its predictions (although any given apparatus can be considered as a quantum system). We investigate the possible alternatives. Theories where the observed world is deterministic but the observer is not (whatever the reason for that) lead to Bell's nonseparability theorem. If, on the other hand, the observer too is deterministic, the theory is not verifiable. It follows that quantum theory must be the logically preferred option. Its inability to completely describe the measurement process appears to be not a flaw of the theory but a *logical necessity* which is analogous to Gödel's undecidability theorem.

I. THE AMBIVALENT OBSERVER

Despite its uncontested success, quantum theory has been considered as somewhat unsatisfactory by many distinguished physicists, including some of its creators.[1–3] The difficulty lies in the description by the theory of experimental observations. Indeed, we know that apparatuses are made of atoms and we known in principle how to analyze their behavior by means of Schrödinger's equation. However, the "collapse of the wave function"—the unpredictable selection of one component of a superposition (or mixture)—cannot be described by Schrödinger's equation, or any other linear equation.

To trace the root of the difficulty, we first note that we have tacitly divided the world in three parts: (a) *the physical system*, the properties of which are being investigated, (b) the laboratory equipment interacting with it (conventionally called "*the observer*"), and (c) *the rest of the world*, which we ignore because we consider it as irrelevant.

The "system" is described by quantum theory, and its dynamical variables form a noncommuting algebra. The various ways to prepare it are represented by rays in a Hilbert space, whereupon the dynamical variables can be represented by linear operators. Consider one such operator, P, with eigenvalues 0 and 1. The measurement of P (that is, a macroscopic procedure ending in "0" or "1," which our rules of interpretation call the "measurement of P") proceeds as follows[4]:

We couple the system, prepared in a state ψ, with the observer, so that together they form a single dynamical quantum system. A suitable Hamiltonian for this composite system might be

$$H = H_{\text{sys}} + H_{\text{obs}} - i\hbar\delta(t - t_0)P\,\partial/\partial\theta, \tag{1}$$

where θ is a dynamical variable of the observer, for example, the angular position of a pointer. Just before the observation time t_0, the observer state ϕ_0 is peaked around, say, $\theta = 0$ and the composite system is in the state $\phi_0\psi$.

Immediately after the observation, the state has evolved into

$$\begin{aligned}&\phi_0(1 - P)\psi + \exp(-\partial/\partial\theta)\phi_0 P\psi\\ &\equiv \phi_0(1 - P)\psi + \phi_1 P\psi.\end{aligned} \tag{2}$$

Note that $\phi_1 = \exp(-\partial/\partial\theta)\phi_0$ is sharply peaked around $\theta = 1$, i.e., the pointer is aiming toward the value 1.[5]

At this stage, it is customary to assert[6–8] that, as the states ϕ_0 and ϕ_1 are macroscopically different, it is impossible to observe their relative phase. Therefore the wave function (2) actually represents a mixture (not a coherent superposition) and can be interpreted as a *classical* ensemble of apparatuses with pointer positions at 0 or 1. On the other hand, it can be argued[8–10] that it is not really impossible to observe the relative phase of ϕ_0 and ϕ_1, that this is only *very* difficult, but that if we try hard enough we should be able to display interference effects between the two components of Eq. (2). In other words, Eq. (2) is *not* the explanation of a measurement, whereby the wave function suffers an irreversible change.

Anyway, even if we accept Eq. (2) as the description of a measurement, it cannot be the entire story. At this stage, we *must* also change the language which we use and convert our quantum observer into a classical observer.[11] However, this "translation" cannot be one to one: The position of the pointer is finally recorded as a single, definite number, not as "either 0 with probability $|(1-P)\psi|^2$ or 1 with probability $|P\psi|^2$." This necessarily imperfect translation is the root of all the epistemological difficulties related to quantum theory. The theory works in a way which is essentially statistical: It predicts the probabilities of various events, under well-defined experimental conditions. But in our consciousness (in the records taken by our apparatuses) there is only a single world. Each event is *unique*.[12]

Could there be a better theory, describing individual events?

II. THREE WISHES FOR A THEORY

The genie, freed from an uncorked bottle, granted us three wishes for a better theory. What could we reasonably request?

A. Determinism

The theory should be deterministic, or at least cryptodeterministic. In principle, the outcome of a perfectly controlled experiment should be predictable. In practice, we may not be able to make full use of this determinism, because of inadequate control of some of the variables (e.g., the position of every atom in the classical kinetic theory of gases) and our predictions may have only a statistical character. However, the underlying reality is fully deterministic.

B. Verifiability

Experimental verification should be contingent. In particular, if several experiments are deemed mutually incompatible by the theory, the observer may perform *any* one of them. Which one is actually performed may be arbitrarily chosen by the free will of a human observer, or by any other random process such as an atomic decay in a computer controlled experiment.

C. Universality

The theory should be universal. In particular, it should apply to any piece of laboratory hardware.

Each one of these three wishes seems reasonable, yet

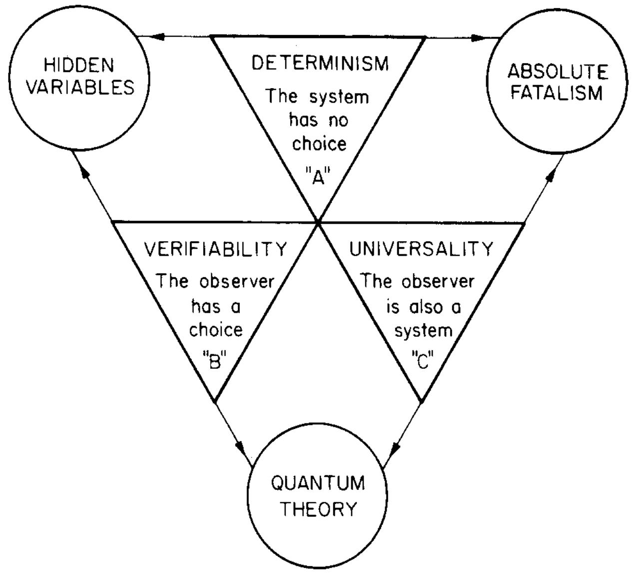

Fig. 1. Three wishes and their consequences.

they are obviously incompatible (see Fig. 1). The remaining sections of this paper consider the fulfillment of any two of the three wishes.

III. A AND B: OBJECTIVE HIDDEN VARIABLES

If observers enjoy the privilege of immunity from the laws of the deterministic theory, we may get a logically consistent scheme[13,14] albeit not a universal one. (For example, celestial mechanics is deterministic and puts no restrictions on our ability to measure the positions of planets and asteroids, but celestial mechanics does not explain the functioning of telescopes and photographic plates, nor was it intended to.)

On the other hand, we may believe that apparatuses are made of atoms and that their macroscopic behavior should be reducible to that of their constituents. There is nothing in quantum theory making it applicable to three atoms and inapplicable to 10^{23}. If we are going to forego the third wish, we should explain what makes observers (i.e., measuring apparatuses) essentially different from other physical objects. As a tentative explanation, it could be argued that since observers are very big, it is impossible to isolate them from the rest of the world.[15] Any information transfer to or from the observer is associated with an energy transfer incomparably larger than the spacing of the observer's energy levels. The evolution of the observer thus is essentially that of an *open* system.[16] It cannot be completely reduced to that of its elementary constituents. Only a microscopic system, having a few constituents, can be approximately isolated from outside influences during at least a finite lapse of time.

This argument, when applied to quantum theory may well contain the clue about the ultimate resolution of the measurement paradox. For instance, one may show that presence of the environment determines a preferred *pointer basis* in the quantum apparatus.[17] The pointer of such an "open" apparatus can be observed only in one of the states of the pointer basis, and not in their superposition. This is a consequence of the interaction between the apparatus and its environment which continuously destroys the relative phase between the states of the pointer basis and "collapses" the state of the apparatus into one of the states of that basis. Of course, any explanation which derives the stochastic nature of observers from their interaction with the environment forces one to reassess the methodology at the very basis of our considerations, namely, the separability of the world into the system, the observer, and the rest of the world.

In fact, nonseparability plagues cryptodeterministic theories in an even more spectacular way. Bell's theorem,[18] which is a formal statement of the EPR paradox,[19] asserts that the outcomes of measurements by one observer cannot be independent from actions taken by another observer, located at an arbitrarily large distance. This result, which has been hailed as "the most profound discovery of science,"[20] deserves a careful scrutiny in the light of the preceding remarks.

There are many ways of proving Bell's theorem[21–23] of which the following is best suited for our purposes: Consider a physical system consisting of two widely separated subsystems, having interacted sometime in the remote past. An observer examining one of the subsystems has *the choice between two mutually incompatible procedures*: he can either measure a variable A with possible results $a = \pm 1$, or a variable C with possible results $c = \pm 1$. A second observer examines the other subsystem and has likewise the *choice* of measuring either B (with result $b = \pm 1$) or D (with result $d = \pm 1$). At the times of these measurements the two observers are unaware of each other, e.g., they are mutually spacelike.

Each one of these observers may thus consider the other one as belonging to the rest of the world and its actions as irrelevant. For example, the result $a = 1$ obtained by the first observer should not depend on whether the second one measures (has measured, will measure) B or D.

There are therefore four possible experiments, namely, measuring A and B, or A and D, or C and B, or C and D. Only one of these experiments is actually performed, but the results of the other ones are quite definite, although they cannot be known to us (this is the assumption of cryptodeterminism). These experiments are repeated many times. For the jth run, we trivially have

$$|a_j(b_j + d_j) + c_j(b_j - d_j)| \equiv 2, \tag{3}$$

and therefore on the average

$$|\langle ab\rangle + \langle ad\rangle + \langle cb\rangle - \langle cd\rangle| \leqslant 2. \tag{4}$$

(This is Bell's inequality.) However, although the *individual* results $a_j,\cdots,d_j$ are unpredictable, their *correlations* $\langle ab\rangle,\cdots,\langle cd\rangle$ are *average* values which can be computed by quantum theory, or any other valid theory, or can actually be measured experimentally, irrespectively of any theory.[23] Now, it is not difficult to prepare situations where the inequality (4) is violated, so that the *identity* (3) cannot be valid. be valid.

This remarkable result is often construed as a proof of violation of separability in the real world. It has even been suggested that there is an advanced potential or a tachyon-like "influence" relating the various observers.[24–28] The most amazing property of this influence is that it affects only individual outcomes, but never averages or correlations. An entirely new kind of physics seems to appear here and has even been invoked in support of parapsychology![29] It is therefore important to remember the precise domain of validity of Bell's theorem: *its proof requires the observed system to be deterministic, while the observer is not.*[30]

IV. A AND C: ABSOLUTE DETERMINISM

Renouncing our second wish (the possibility of choosing experiments) makes the world completely incomprehensible. To analyze an experiment which we perform, we would first have to analyze our own brain. As stated by Bell,[31] "separate parts of the world would be deeply and conspiratorially entangled, and our apparent free will would be entangled with them." This "totalitarian theory" has, however, one advantage—we cannot write the troublesome Eq. (3) because only one of the four different experiments can be performed and, given all the necessary initial data, it is the theory itself which determines which one. The other three experiments are not legitimate alternatives.

Paradoxically, we also have kind of complete determinism in Everett's "relative state" interpretation[32,33] of quantum theory, which is the statistical interpretation[34] taken literally: The wave function describes an infinite multitude of noninteracting worlds, and everything which may happen, does happen. The wave function never collapses, but some of the worlds bifurcate and evolve in different ways. (Determinism of course holds only for the super-Universe including the totality of these worlds, not for each world separately.) Here too Eq. (3) causes no difficulty. All four experiments are performed in some worlds, but incompatible experiments happen in different worlds. In particular the a_j in the worlds where we measure A and B need not be the same as the a_j in the worlds where replicas of ourselves measure A and D.

The difficulty in Everett's theory is not its extravagant profligacy (which costs nothing) but the ambiguity in what constitutes *different* worlds. The arguments and counter-arguments following Eq. (2) now acquire a crucial importance. In the absence of a precise criterion to distinguish experimentally a coherent superposition from a mixture, it is impossible to decide whether or not the world has split.

V. B AND C: QUANTUM THEORY AGAIN

By now, the difficulties which led us to wish "better" theories may appear less terrible than their alternatives. In this section, we show how a consistent scheme can be developed in spite of the elusive nature of the observer. The point is that any apparatus (designed to function as an observer) *can* be described by quantum theory and made to interact in the fashion described by Eq. (1). However, this quantum description is valid only when the system to which it is applied is considered isolated. Moreover, applied to the apparatus it allows a superposition of different apparatus states and so precludes this apparatus from acting as the observer.[17] Such an apparatus can be then observed by another apparatus, for which the theory gives *no dynamical description*, only the probability rules.

It does not matter whether the "true observer" is a grain of silver bromide, or Schrödinger's cat,[3] or Wigner's friend[35] or Wigner himself observing his friend, as long as this observer is *not* described by quantum theory and, in particular, is not in a superposition of macroscopically different states. Nevertheless, quantum theory *is universal.* It applies to the grain of silver bromide, to the cat, etc., but under the condition that they lose their status of observers and have to be observed by something or someone else. Equations of quantum theory do not suggest any hierarchy of observers; in the EPR situation,[19] each observer can consider the other one as part of his "apparatus."

Thus although quantum theory is universal, it is not *closed*. Anything can be described by it, but something must remain unanalyzed. This may not be just a flaw of quantum theory: It is likely to emerge as a logical necessity in any theory which is self-referential, as it attempts to describe its own means of verification. In this sense it is analogous to Gödel's undecidability theorem[36,37] of formal number theory: the consistency of the system of axioms cannot be verified because there are mathematical statements which can neither be proved nor disproved by the use of the formal rules of the theory, although their truth may be verified by metamathematical reasoning.

Finally, let us examine the status of the troublesome Eq. (3) in a theory where individual outcomes are genuinely random and not due to some underlying deterministic reality. In that case, it is not legitimate to mix in the same equation the results of real experiments and those of counterfactual ones. The mathematical symbols representing the latter are devoid of any meaning. These unperformed experiments have *no* results.[38–40] Of course, nobody can prevent us from dreaming about their results, had they been performed; but then, in such a dream, there is no reason to request that the correlations of Eq. (4) be those of quantum theory, since quantum theory itself precludes observing these correlations. Therefore there is no contradiction possible, nor any proof of nonseparability.

VI. CONCLUSION

In Sec. I of this paper, we listed specific complaints against quantum theory. In Sec. II we were given the freedom to imagine a "better" theory. The alternatives, discussed in Sec. III and IV, turned out loaded with conceptual difficulties even more serious than those of quantum theory itself. We then returned to quantum theory in Sec. V and we conclude that although it can describe *anything*, a quantum description cannot include *everything*. Whenever we place the interface between the quantum world and our experimental records, this interface is either an abstract probability rule with no physical description, or it must have *two incompatible descriptions*, one quantized and one classical, and it is the "translation" between them which is probabilistic.

ACKNOWLEDGMENTS

We are grateful to J. A. Wheeler for many stimulating discussions. This work was supported by NSF Grant PHY-7826592.

[a] Permanent address: Technion–Israel Institute of Technology, Haifa, Israel.

[b] Present address: Theoretical Astrophysics 130-133, California Institute of Technology, Pasadena, CA 91125.

[1] A. Einstein, in *Albert Einstein, Philospher–Scientist*, edited by P. A. Schilpp (Library of Living Philosophers, Evanston, IL, 1949).

[2] L. de Broglie, *Une tentative d'interprétation causale et non linéaire de la mécanique ondulatoire* (*la théorie de la double solution*) (Gauthier-Villars, Paris, 1956).

[3] E. Schrödinger, Naturwiss. **23**, 807, 824, 844 (1935).

[4] J. von Neumann, *Mathematical foundations of quantum mechanics* (Princeton University, Princeton, NJ, 1955).

[5] Here, we have tacitly assumed that the wave funcitons ϕ_0 and ϕ_1 mean that the pointer is *objectively* at the positions $\theta \simeq 0$ and $\theta \simeq 1$. That is, the measuring apparatus need not be measured by a further apparatus, and so on *ad infinitum*.

[6] A. Daneri, A. Loinger, and G. M. Prosperi, Nucl. Phys. **33**, 297 (1962); Nuovo Cimento B **44**, 119 (1966).

[7] K. Hepp, Helv. Phys. Acta **45**, 237 (1972).

[8] A. Peres, Phys. Rev. D **22**, 879 (1980).

[9] J. S. Bell, Helv. Phys. Acta **48**, 93 (1975).

[10] P. Jasselette, Int. J. Quantum Chem. **17**, 83 (1980).

[11] A classical description is *not* an approximation of a quantized description, but is *qualtitatively different* (see Footnote 5 above). A classical pointer has a position which is a real number. A quantum pointer also has a position, but the latter is an operator. There are questions, such as "what is the numerical value of the pointer position," which make sense in the classical language, but are meaningless in the quantum language. (The following analogy might help to understand this translation problem. The laws of gases are qualitatively different from the Newtonian laws of motion of 10^{23} molecules. There are statements about temperature, entropy, etc., which make sense in the thermodynamic language, but not in a complete description of 10^{23} individual molecules.) The only vestige of quantum theory which a classical description retains is the uncertainty principle, namely, a spread of order $\hbar^n$ in the $2n$-dimensional phase space.

[12] Even if we repeat the "same" experiment many times, each trial may be given a different serial number.

[13] J. S. Bell, Rev. Mod. Phys. **38**, 447 (1966).

[14] D. Bohm and J. Bub, Rev. Mod. Phys. **38**, 453 (1966).

[15] H. D. Zeh, Found. Phys. **1**, 69 (1970).

[16] No computer, *a fortiori* no human, can function in thermodynamic equilibrium. See I. Prigogine, Science **201**, 777 (1978).

[17] If we succeed in including the apparatus in the wave function and there is nothing else, we get a continuous, reversible evolution which is *not* the description of the measurement (see discussion at the end of Sec. I). However, continuous interaction of the apparatus with the enviroment can result in the apparent collapse of the state vector: W. H. Zurek, Phys. Rev. D **24**, 1516 (1981); see also W. H. Zurek in *Quantum Optics, Experimental Gravitation and Measurement Theory*, edited by P. Meystre and M. O. Scully (Plenum, New York, 1982).

[18] J. S. Bell, Physics **1**, 195 (1964).

[19] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[20] H. P. Stapp, Nuovo Cimento B **29**, 270 (1975).

[21] H. P. Stapp, Phys. Rev. D **3**, 1303 (1971).

[22] P. Eberhard, Nuovo Cimento B **38**, 75 (1977).

[23] J. F. Clauser and A. Shimony, Rep. Prog. Phys. **41**, 1881 (1978).

[24] H. P. Stapp, Nuovo Cimento B **40**, 191 (1977).

[25] O. Costa de Beauregard, Nuovo Cimento B **42**, 41 (1977).

[26] M. Slaby, Bull. Am. Phys. Soc. **23**, 586 (1978).

[27] J. G. Cramer, Phys. Rev. D **22**, 362 (1980).

[28] S. J. Feingold and A. Peres, J. Phys. A **13**, 3187 (1980).

[29] O. Costa de Beauregard, Phys. Lett. **67** A, 171 (1978).

[30] Bell himself never claimed more than he actually proved, namely, the nonseparability of objective hidden variable theories. Any misrepresentation of his theorem, implying the physical existence of a superluminal influence, is due to the careless analysis of other authors.

[31] J. S. Bell, J. Phys. C **2**, 41 (1980).

[32] H. Everett III, Rev. Mod. Phys. **29**, 454 (1957).

[33] B. S. De Witt and N. Graham, *The many-worlds interpretation of quantum mechanics* (Princeton University, Princeton, NJ, 1973).

[34] L. E. Ballentine, Rev. Mod. Phys. **42**, 358 (1970).

[35] E. P. Wigner, in *The scientist speculates*, edited by I. J. Good (Basic, New York, 1962).

[36] K. Gödel, Monat. Math. Phys. **38**, 173 (1931).

[37] E. Nagel and J. R. Newman, *Gödel's proof* (New York University, New York, 1958).

[38] J. A. Wheeler, in *Mathematical foundations of quantum theory*, edited by A. R. Marlow (Academic, New York, 1978).

[39] A. Peres, Am. J. Phys. **46**, 745 (1978).

[40] A. Berthelot, Nuovo Cimento **57** B, 193 (1980).