INDETERMINISM IN QUANTUM PHYSICS AND IN CLASSICAL PHYSICS [1]

PART I

I

BEFORE entering into a more detailed discussion, I shall attempt, in this introductory section, to state my main points in outline.

Quantum physics is now generally admitted to be indeterministic in the sense that it implies the impossibility of predicting certain kinds of physical events, however complete our initial information may be concerning the physical system in question ; given sufficiently precise initial information we may, however, predict the probability of these events, i.e. the frequency of their occurrence under sufficiently similar conditions. Classical physics, on the other hand, is usually taken to be deterministic in the sense that it implies the predictability, with any desired degree of precision, of every single physical event, on the basis of sufficiently precise initial information. In the present paper I propose to show that the opposition indicated here is misleading even although the *prima facie* deterministic character of classical physics must be admitted. In spite of important differences, the situation in classical physics shows greater similarities to that in quantum physics than is usually believed. My thesis is that most systems of physics, including classical physics and quantum physics, are indeterministic in perhaps an even more fundamental sense than the one usually ascribed to the indeterminism of quantum physics (in so far as the unpredictability of the events which we shall consider is not mitigated by the predictability of their frequencies).

The impossibility, implied by quantum physics, of predicting events of a certain kind is an impossibility of a peculiar character. If we assert of an observable event that it is unpredictable we do not mean, of course, that it is logically or physically impossible for anybody to give a correct description of the event in question before it has occurred; for it is clearly not impossible that somebody may hit upon such a description accidentally. What is asserted is that certain rational

[1] Expanded version of a paper read before the Philosophy of Science Group of the British Society for the History of Science, at their first Ordinary Meeting on November 15th, 1948.

methods of prediction break down in certain cases—the methods of prediction which are practised in physical science. These methods involve various physical processes, among them those of obtaining initial information by way of observation. It is the analysis of these latter processes in the light of quantum physics which establishes the impossibility of obtaining predictions in the cases in question. The impossibility asserted is therefore a physical impossibility—that of successfully carrying out certain physical operations involved in obtaining predictions in accordance with the methods of science.

It is usually asserted (I think correctly) that the physical processes involved in observation lead to difficulties only in quantum physics, and that they disappear if it is assumed that Planck's quantum of action equals zero—in other words, in classical physics. Nevertheless I contend that if, in a similar way, we analyse besides the physical processes involved in observation certain other physical processes which are also involved in every prediction, then we find a somewhat similar result which, however, is valid for classical physics also. The processes which our analysis must take into account besides observation are the physical processes involved in the calculation and formulation of the predictions. If this is done then we find that all scientific predictions are in an important sense deficient, including the most complete set of predictions that is physically possible to formulate, even from the point of view of classical physics; and we find that this deficiency becomes significant whenever we wish to predict the behaviour of physical systems (classical or otherwise) of a certain kind, viz, of predicting machines.

Our procedure will be to consider some of the properties, and especially the limitations, of what we shall call a 'predictor', i.e. a classical mechanical calculating and predicting machine which is so constructed as to produce permanent records of some kind (such as a tape with holes punched into it) capable of being interpreted as predictions of the positions, velocities, and masses of physical particles. It will be shown that such a machine can never fully predict every one of its own future states, and consequently not those of what we shall call its (closer) 'environment', i.e. the part of the world with which it (strongly) interacts. Such a machine, moreover, can never fully predict, or be predicted by, any sufficiently similar machine with which it interacts. Thus if we call a set of similar and interacting predictors a 'society', then we can say that no member of such a society can fully predict the future states of that society, or those of any of its members.

All this follows very simply from the general idea of a physical predictor. But if we further take into account the implications of either classical particle mechanics or of certain other classical systems of physics then we even find that the following alternative holds for every classical physical world. Either no predictor exists in the world in question, and consequently no predictions in a physical sense, or the future states of one at least of the existing predictors cannot be predicted by any of the existing predictors.

But this implies the physical impossibility of predicting, with the help of the methods of science, certain physical events ; or, in other words, it implies an indeterminism of a kind somewhat similar to the one implied by quantum physics although, it seems, unmitigated by the predictability of probabilities or frequencies.

Our argument is somewhat similar to certain ideas of Niels Bohr's who suggests repeatedly[1] that there is an 'impossibility of distinguishing, in introspection, sharply between subject and object', and who connects this idea with indeterminism ; 'it must never be forgotten', he writes,[2] 'that we ourselves are both actors and spectators in the drama of existence.' Bohr suggests that these considerations may be applicable to psychology, and that a kind of 'complementarity' may exist between explanation and volition. Now our physical predictors, too, are actors and spectators, as it were. They compute ; and what they compute are descriptions. Nevertheless, our argument does not appear to be one of involving what Bohr has called 'complementarity'.[3] For complementarity is a symmetrical relation (if *a* is complementary to *b*, then *b* is complementary to *a*). Admittedly, the fact that a predictor can predict a 'society' only if it does not become a member of it, has some resemblance to complementarity. But the more fundamental fact that a predictor cannot *predict* itself (although it may *explain*, as we shall see, its own actions after the

[1] See for example, Niels Bohr, 'Biology and Atomic Physics', *Celebrazione del Secondo Centenario della Nascita di Luigi Galvani, Congressi Scientifici*, Bologna, 1938, p. 14 ; and 'On the Notions of Causality and Complementarity', *Dialectica*, 1948, 2, 318. See also Gilbert Ryle, *The Concept of Mind*, London, 1949, pp. 195 *et seq.*

[2] Bohr, *loc. cit.* (*Dialectica*).

[3] To Bohr's distinction between ourselves as actors and as spectators (of ourselves) corresponds Ryle's distinction between ourselves as 'performing' and as 'commenting' (upon our performance). But this relation is not a complementary one since it is not symmetrical but hierarchical in character ; to comment upon a performance 'is to perform a higher order act'. (Ryle, *loc. cit.*, p. 195.)

event) does not appear to point to any other limitation which may be said to be complementary to it.

2

We shall take indeterminism to be a doctrine asserting that *not all* events are 'determined in every detail' (whatever this may mean), and determinism as asserting that they *all* are, without exception, whether future, present, or past. It is necessary to be clear that the problem contested by the two doctrines is only [1] this, and that indeterminism does not, perhaps, assert that all or most or many events are not determined, but only that some events which are not completely determined exist—however rare they may be.

How the word 'determined' should be interpreted in this context is not quite easy to say. In fact, there exist a good number of possible interpretations and, accordingly, a number of different doctrines of determinism (and, of course, of indeterminism). Although we shall discuss in detail only one of these—the one which interprets the word 'determined' as 'predictable in accordance with the methods of science'—it may be of interest first to refer briefly to some of the other doctrines.

One can present the general idea of determinism as arising out of a critical revision of a commonsense view of the world—the view that some future events or occurrences (such as the apparent daily or yearly movements of the sun) are 'necessary' or 'predetermined' and therefore predictable, while others (such as the vagaries of the weather) are 'due to accident' and therefore unpredictable. A slightly more sophisticated view arises out of the doubt whether this classification is a valid one; whether the apparently 'accidental' events do not appear to us in this way merely because of our lack of knowledge; and whether we should not be able to predict them like the others if only we knew more about the conditions of their occurrence.

When this more sophisticated view develops into a doctrine, it may be called the 'metaphysical doctrine of determinism'. According to this doctrine, all events of this world are completely predetermined. While, to the commonsense view, there is a difference in the degree of 'fixedness', or alterability, between future and past

[1] This is, for example, clearly expressed in P. W. Bridgman, *The Logic of Modern Physics*, New York, 1927, p. 210; while any such reference to *all* events is missing in the description of determinism in Max Born, *Natural Philosophy of Cause and Chance*, Cambridge, 1949, pp. 8 and 9. (See also the next note.)

events—the future is open, as it were, and future events are still alterable, while the past is unalterably fixed—metaphysical determinism believes that future events are, even before they happen, unalterably fixed, in exactly the same sense as past events are fixed. Accordingly, they might be foreknown—in exactly the same sense as they might be known after they have happened. Whether or not anybody has any such foreknowledge of them or any historical knowledge after the event, is, of course, a different question.

But it ought to be realised that, should anybody possess any real foreknowledge of an event—not a mere guess—then this would imply that the event was predetermined. Accordingly, any doctrine asserting the existence or possibility of complete [1] foreknowledge implies the metaphysical doctrine of determinism. As J. S. Mill correctly observed, the doctrine of an omniscient being implies, for this reason, that of determinism (if by 'omniscience' we mean the ability to answer with certainty every conceivable question, including questions concerning the future). For the question whether an event of a certain description will happen at a future date can be answered now with certainty only if it is now certain whether this event will or will not occur. It may be remarked that for this reason, the doctrine of omniscience contradicts that of omnipotence ; even if we take this latter doctrine in the very weak sense as merely implying that if an omnipotent being exists now, it has now some power, i.e. it can now alter the future course of events. For if we assume that the future course of events is now predetermined, then we also must assume that it is strictly impossible now to alter the course of events, and that every being is now completely powerless to achieve such a feat.

The metaphysical doctrine of indeterminism merely asserts that there exists at least one event (or perhaps, one kind of events, such as certain human activities, and events depending on them) which is not predetermined ; or that there is at least one question about the future which cannot be answered with certainty but must be 'left open'.

Both the metaphysical doctrines of determinism and indeterminism are, clearly, not testable. For even if the future would constantly

[1] A doctrine (such as Born's) which merely asserts that 'events at different times are connected by laws in such a way that predictions of unknown situations . . . can be made', without asserting that this holds, in principle, for *all* events, or all unknown situations, does not, of course, assert the possibility of *complete* foreknowledge.

surprise us, and show no sign of any predetermination, it might still be predetermined, and even foreknown—written in the book of destiny. On the other hand, even if everything had a completely regular and deterministic appearance, this would not establish that there is no single undetermined event. But lack of testability (in my opinion[1]) does neither here nor elsewhere establish lack of meaning, although it may be taken to establish lack of what might be called 'empirical content'. A doctrine may be logically too weak to be tested, and may yet be implied, just because of its weakness, by a testable doctrine. This contingency appears to be realised, more or less, in our case ; for the doctrine of determinism may be given a testable form, if we interpret determinism by scientific predictability. And it may be claimed that, in this form, it implies the metaphysical doctrine of determinism in the same way in which any doctrine of foreknowledge implies this metaphysical doctrine.

3

The *prima facie* deterministic character of classical Newtonian mechanics is perhaps most strikingly illustrated by the story of the so-called '*Laplacean demon*'.

The 'Laplacean demon' is a superhuman intelligence capable of knowing the positions, masses, and velocities of all elementary particles (point-masses) for a certain moment of time t_0. Laplace pointed out that this knowledge (we shall call it 'initial information') of the t_0 state of the world together with a knowledge of Newtonian theory of mechanics would enable the demon to predict, by way of logical or mathematical deduction, every future state of the world. (It is thereby assumed that the problem of more than two gravitating bodies is soluble.)

One can argue that this kind of predictability is deterministic—even in a stronger sense than is the doctrine of metaphysical determinism. For since the Laplacean doctrine implies that a knowledge of present or past states involves the foreknowledge of any future state, all future states must be determined now. (Or at least, this appears to be implied by the laws of classical mechanics.)

The Laplacean doctrine of determinism replaces the vague idea of unalterability of fixedness by the much more precise idea of *predictability in accordance with a definite rational method*—the method of

[1] Cf. my *Logik der Forschung*, Vienna, 1935, pp. 8 *et seq.*, 33, 74

science which deduces predictions from initial information in connection with theories. This, no doubt, is an important improvement on the metaphysical doctrine. But the Laplacean doctrine is still too abstract to be testable. It demands an amount of initial knowledge which may turn out to be unobtainable since it may be infinite in several respects. For the doctrine speaks of the whole world which may contain an infinite number of particles ; and it assumes infinitely precise measurements.

One might say that all these difficulties arise from the fact that the story of the Laplacean demon is an attempt to eliminate the vague and dangerous phrase 'in principle'. For what it tries to explain is what we mean when we say that the future states of a system can be 'in principle' predicted on the basis of a knowledge of past or present states. 'In principle' means here something like 'not in practice, because human knowledge is never sufficiently precise and complete'. No wonder that, in attempting to explain what we mean by 'in principle', Laplace was driven to introduce a superhuman intelligence. But the Laplacean demon is unsatisfactory, we may say, just because infinitely precise and complete knowledge is also 'in principle' unattainable.

However, it appears that we may avoid the infinities and impossibilities involved in the Laplacean doctrine, and formulate a *finite version* of the doctrine of determinism.

Several steps must be taken in order to achieve this. The most important of them is to *embody* Laplace's disembodied spirit, that is to say, to make this spirit *a member of the physical world which it tries to predict.* We shall consider, accordingly, a 'predictor' instead of a demon, i.e. a physical predicting machine. And we shall assume, in discussing classical mechanics, that this machine is a classical mechanism, subject to the laws of mechanics.

Such a machine will be able to obtain measurements of the masses, positions, and velocities of the physical particles within a certain finite spatial region—its 'environment'—for example, by using sensitive tentacles, and measuring rods, etc. It will be able to carry out a number of physical operations which we may interpret as calculations proceeding on this initial information in conjunction with physical theories and mathematical methods which, we may assume, are part of the total information with which the calculator is supplied. (This part of the information, we may assume, was built into the calculator by its makers.) And it will be able, ultimately, to produce predictions in the form of some permanent record such as a punched tape (or a typewritten report).

The part of the world which such a predictor can explore—its environment—must be finite. Accordingly, the predictor will be unable correctly to predict events within its environment which it might have predicted on the basis of information about conditions beyond its environment (we can call such events ' due to interference from outside '). Similarly, the measurements it can take, and the predictions it can produce, will be at best of a certain finite degree of precision. As a consequence, we can expect that predictions about the near future will be more successful than predictions about the more distant future.

We can now introduce the auxiliary idea of a '*specified finite prediction task*'. By this we mean *the task of predicting, with some chosen and specified degree of precision, some event—i.e. the position and/or velocity of its particles—occurring in a finite mechanical system, sufficiently isolated from outside interference, at a certain chosen future instant of time.*

We can make this idea more definite still, for example, by specifying limits to the period of time within which the instant may be chosen for the event to be predicted. We may also, perhaps, specify an upper limit for the size of the physical system (especially if we are interested in such special problems as that of human determinism). And we may, moreover, specify more precisely that, and under which circumstances, an unsuccessful prediction is not to be counted as a negative instance if we find evidence, after the event, that it may be accounted for (on the basis of the theories used) as due to interference from outside, or in other words, to insufficient isolation. However, the problem of interference from outside shall not be considered here in more detail.

With the help of this, we may now formulate our finite version of the deterministic doctrine.

For any specified finite prediction task, it is physically possible to construct a predictor capable of carrying out this task. (' Physically possible ' means ' possible from the point of view of the system of physics under consideration '—for example, classical mechanics.)

Since, for every specified degree of precision of the prediction, there will be a corresponding (often higher) degree of precision of the necessary initial measurements, this formulation implies that it is physically possible to construct a predictor capable of obtaining initial information with any specified finite degree of precision (i.e. short of absolute precision).

The finite version here given is an attempt to formulate that doctrine of determinism which is rejected by quantum physics, and which, as I shall try to show, is also incompatible with most classical systems, for different although perhaps somewhat similar reasons.

4

Our formulation of the finite doctrine refers essentially to some system of physical theory. Such a system is assumed to be incorporated in the predictor ; besides, it must be used when we construct a predictor capable of carrying out the specified task. (And ultimately, it must be used when determining whether the degree of isolation of the system was sufficient.)

It may be thought that this reference to a certain theory is unnecessary, and that an alternative finite formulation may be given which is free from it, as follows : The states of two sufficiently isolated physical systems whose states at the instant of time t_0 agree with a sufficient degree of precision will, at any later instant t_1, agree with any specified degree of precision.

A very similar further alternative would be the formulation: every physical event in a system can be reproduced, by way of reproducing in another system one of the states which preceded the event in question.

Both these formulations appear to be independent of any reference to a physical theory ; but this appearance is misleading. For the ideas of a physical event, and of a physical state, etc., are dependent upon the theory chosen. (For example, whether an event or state may be characterised by stating, apart from positions, the velocities, or whether time derivatives of a higher order will have to be obtained, depends upon the theory—whether its differential equations are of the second or a higher order.) To put it in another way : without theory we can have no idea whether two systems which ' look ' very similar are physically similar or not, and therefore, what the degree of their similarity is.

We could of course always say, if the two systems turned out to become obviously dissimilar in the course of time, that they were not in the same state to start with. But this would mean *assuming* a deterministic principle ; and in such a way that it could never be tested.

An example may be helpful. We may build two clocks so similar that we cannot detect a difference. We shall expect, nevertheless, that they will show a deviation after the lapse of some time. If they can be ' regulated ' by some mechanism, we can reduce this deviation—but as a rule at the cost of introducing a clearly visible difference into the position of the two regulator mechanisms. Thus, in practice, we do not proceed in the way described by our alternative formulations ; but we proceed in full accordance with our theories which make us expect that the degree of similarity of the two clocks will be insufficient for certain purposes.

If, however, we keep in mind that the two alternative formulations given in this section are to be taken as relative to a theory which determines what is, and what is not, a similar state, then we can accept them as equivalent to our previous finite version, provided we remember that this similarity is to be determined by a physical predictor. This would lead to a reformulation of our latest version along lines such as these :

For every physical event, there exists a predictor (i.e. it is physically possible to construct a predictor) capable of reproducing the event in another system by way of reproducing one of the states which preceded the event in question.

This formulation extends the idea of a ' predictor ' to that of a ' reproducer ', but I do not think that this extension means an important alteration in our approach.[1]

5

Our finite version, by postulating the existence of a predictor (i.e. the physical possibility of its construction) capable of carrying

[1] I do not discuss a very different doctrine of determinism which may be called ' programmatic determinism ' and which may or may not be connected with a belief in the metaphysical doctrine. I have in mind the doctrine (which, it appears, is held for example by Einstein) that, even if our present physical theories do not support determinism, we shall advance, in time, to the discovery of a completely deterministic system of physical theories. Yet it is not, I believe, this programme, but rather the alleged deterministic character of definite theories (such as classical particle mechanics) which creates a serious problem of the philosophy of science ; only the belief that such physical theories actually exist, and that they have been successful, gives plausibility to the hopes of the programmatic determinist. (That this fact is neglected, and that his criticism confines itself to what I call ' programmatic determinism ', is the reason why I do not think that Professor Ryle's otherwise brilliant comments on the problem are completely satisfactory ; see Gilbert Ryle, *The Concept of Mind*, pp. 76–81.)

out prediction tasks with any specified precision, implies that every predictor can be improved, in the sense that it is always physically possible to construct one whose measurements are more precise than any given one. Or, in other words, predictors may be constructed which approximate more and more closely to a perfect predictor capable of absolutely precise measurements.

In connection with later discussions, it is important to realise two points, viz. (1) that this series of predictors is in a definite sense convergent, in so far as their achievements—the degree of precision—converges; (2) that our postulate does not imply the actual co-existence of this infinite series of predictors, but only the possibility of constructing one for every given prediction task.

6

So much about the finite version of the deterministic doctrine which we shall attempt to refute. We now turn to its refutation.

Quantum physics implies the denial of what I have called the finite version. It implies that, however full and precise the initial information obtained, and however well isolated the system in question, there are certain physical events which cannot be predicted, although it is possible to predict the frequency of their occurrence under like conditions. Thus certain finite specified prediction tasks cannot be carried out; and the most perfect predictor which can be constructed cannot reproduce every event, even although it can reproduce every system in such a way that the two systems will coincide in respect of the frequencies of the occurrence of the event in question.

Quantum physics tries to elucidate this result by the assertion that the system, although isolated from outside interference, cannot be completely isolated from what I shall call 'interference from within', viz. from the very centre of the predictor's environment—from the predictor itself. In obtaining initial information, the predictor must interact with the system in question, and this interaction introduces into the system a disturbance whose magnitude is unpredictable within a certain halo of uncertainty related to Planck's quantum of action h. The uncertainty relation can be written

$$\Delta q \Delta p \geq h/4\pi$$

where Δq and Δp are uncertainties of the position and momentum. From this we can see that the uncertainty becomes insignificant for sufficiently great masses. We see also that, if we assume that no quantum of action exists—or in other words, if we assume that $h = 0$—then the uncertainty disappears. Now this last assumption—the absence of a quantum of action—is the distinguishing characteristic of classical physics from the point of view of quantum physics. This is why quantum physicists usually assert that classical physics is deterministic. And indeed, it does not know this kind of indeterminacy.

But I shall try to show that classical physics (and indeed, most if not all systems of physics) knows a similar kind of indeterminacy, also due to 'interference from within.' It may be formulated in this way.

Although, in the absence of Planck's quantum of action, the interference between a predictor P^+ and most systems may be made as small as we like, this is not true for all systems, and especially not if the system under consideration is, in its turn, a predictor P, attempting to predict P^+. Nor is there any reason to believe that it is physically possible, from the point of view of classical mechanics, to construct for every predictor P a predictor P^+ sufficiently superior to be capable of predicting P—not even if we assume (as we did in section 5) that for every predictor, however good, it is possible to construct a better one.

The system which P^+ cannot predict need not be a predictor P. In general, that part of the environment of P^+ with which P^+ may, by its construction, strongly interfere, will be unpredictable for P^+ because, as we shall see, P^+ cannot predict every one of its own future states. Nevertheless P^+ may be constructed, *in the cases ordinarily considered,* in such a way that it cannot interfere strongly with the system under consideration; or more precisely, in the absence of h, P may be so constructed as to interfere as little as we like, with an 'ordinary' system. Our thesis is, simply, that even classical mechanics implies that there are limits to this procedure, and that there exist finite specified classical mechanical prediction tasks which no classical mechanical predictor can perform.

This is our thesis. We proceed to establish it.

7

We shall first discuss the situation in a general way.

Consider a mechanical system A, not containing a predictor, and a predictor B attempting to predict A. This it can do only if (1) B can calculate the results of its interference with A or if (2) B interferes sufficiently weakly with A.

As to (1), B can assess the way in which it interferes with A either by studying its interfering parts B' and their interaction with A—which means that it has to study the system $A + B'$ instead of A—or on the basis of predictions about itself (without splitting off some of its parts). The first of these two alternatives does not help, for in order to obtain the necessary information about $A + B'$, the whole problem arises again. The second alternative also does not help; for it will be shown in the next section (8) that a predictor cannot have such knowledge about itself.

Thus we must consider the alternative (2). This means, in effect, that B is so constructed as to interfere only very weakly with systems such as A; and it can 'know' this fact, as it were, by implication; for example, by being so constructed that its predictions of A neglect its interference with A.

Thus B interferes only weakly with A. On the other hand, A must, under certain circumstances, interfere strongly with B. It must do so because some minute differences at the instant of time t_0 may, in a mechanical system such as A, give rise to considerable differences at the instant of time t_1; but since B is a predictor trying to predict the t_1 state of A, it must be able to react in very different ways to the two minutely different states of A.

We can express this situation metaphorically by saying that we can consider A to be separated from B by something like a 'one-way membrane' which allows only weak influences to pass from A to B, but strong ones in the other direction. We can also say that, if B is a predictor, it must be capable of amplifying certain weak effects reaching it from A. Or in other words, a predictor B must be an amplifier simply because some small differences in the system A may become amplified in A itself in the course of time; and the predictor B must be capable of producing a corresponding amplification.[1]

This description of ours, of the way in which A and B interact, may be said to be undertaken from the point of view of a second predictor C which studies this interaction, i.e. the system $A + B$. C can predict, by measuring the state of A and of B, not only future states of A but also the predictions of that state (the tape) which B will produce; and if C is a better predictor than B, it may even be

[1] The argument is intended to show that predictors must be amplifiers *qua* predictors. But it might also be shown, I believe, that they must be amplifiers merely as recorders of measurements; for they must be capable of recording minute differences by the same method (such as the punching of holes) as large ones.

able to predict how much B interferes with A, and perhaps the extent to which this gives rise to errors in the predictions produced by B.

All this will be possible only if C interferes weakly with the system $A + B$, or in other words, if there is again something like a 'one-way membrane' between $A + B$ and C, which allows strong influences to pass in only one direction—from $A + B$ to C.

Now we assume that B is similar to C in a degree sufficient to reciprocate C's interest in B, i.e. that B begins to study the state of C in a way similar to that in which C is studying the state of B. Then B will amplify certain influences coming from C, just as C amplifies certain influences coming from B. Accordingly, the 'one-way membrane' between B and C breaks down, and with it the conditions for successful prediction. Neither can B predict C, nor can C predict B.

This simple situation is of decisive significance for our problem. It shows that if the system to be predicted has certain characteristics—in the main, that it amplifies the otherwise weak disturbances which it suffers from the predictor—then the predicting task cannot be carried out.

If we analyse this situation more clearly, then we find that it depends on the fact (to be discussed in the next section) that neither of the machines can have knowledge of its own state before that state has passed; in fact, it can obtain information about its own state only by way of studying the results obtained by another machine, or by being given these results (say by way of a tape which may pass from one machine to another). In our case, this means that B, for example, can know about its own state only by studying the state of C, or by being interfered with by C's tape. B, therefore, cannot allow for the influence of its own state upon C for the purpose of predicting C; and *vice versa*.

All this holds, of course, not only for B and C but for any set of sufficiently similar and interacting predictors—for a 'society' of predictors, as we called such a set in section 1. None of the members of such a society can predict the future state of its members in every detail; and accordingly, none can predict the future state of the physical system of which it is a part—of the 'society.'

Our analysis of the system $B + C$ can be said to proceed from the point of view of an outsider, the predictor D. As long as D is outside the system—that is to say, as long as D is in the rôle of a demon—it may predict precisely the way in which B and C disturb each other, and in which they fail accordingly to produce successful predictions. But once *B* or *C* or both ' discover ' *D* (or the instruments used by *D* for measuring the system *B* + *C*), and try to predict it, *D* will be unable to predict the system *B* + *C*, since the future behaviour of the system *B* + *C* will become dependent upon the behaviour of *D* which *D* itself cannot predict ; thus *D* will become a member of the ' society ' to which *A* and *B* belong.

Now it is, of course, of the greatest importance to realise under which circumstances *D* can remain ' outside ' of the society of *A* and *B*, and under which circumstances it becomes a member of this society.

If, as we assumed, *B* and *C* are mechanical systems, then *D* may, for example, remain a ' demon ' relative to *B* and *C* if it is an electro-magnetic predictor, observing *B* + *C* by, say, optical means, or by radar (whose disturbing mechanical effects may be made as small as one likes, from the classical point of view). It then may possibly possess no moving parts, so that *B* and *C*, when studying D, will both simply predict that *D* does not move. These predictions can be easily anticipated by *D* when it has found out the mechanism of *B* and *C*. (D operates electro-magnetically, but is in possession of the theory of mechanics.) *D*, as we have described it here, is a kind of realisation of the Laplacean demon, in so far as it really does not belong to the ' world ' which it predicts ; it transcends the mechanical world. Such a realisation can, however, exist in a physical sense, only because the system of classical mechanics which it transcends is physically incomplete. In other words, were classical mechanics capable of explaining electro-magnetism as due to the movements of mechanical particles, then an electro-mechanical *D* would lose the character of being a demon.

Thus *D*'s capacity of predicting *B* + *C* is due to the fact that classical mechanics is insufficient as a theory of the physical world, and that a purely mechanical determinism, accordingly, is refutable. (There are miracles in a purely mechanical world, as it were,—electro-magnetic miracles.)

We may ask whether *D* might not be a purely mechanical predictor and still remain ' outside ' the society of any *B* and *C*—especially in view of section 5 where we have postulated that there exists, to every predictor, a superior one. The answer to this question turns out to be negative, even though it must be admitted that there are degrees in the membership of a ' society ' and that *D* might be a kind of semi-member of *C* and *B*—sufficiently superior in its structure to be a little more successful in predicting *C* and *B* than these are in predicting *D*. (That such 'degrees' of membership are possible follows from the fact that *D* may be a mixed electrical and mechanical system.)

Upon the assumption of determinism (section 3) that to every prediction task there exists an appropriate predictor, there must exist mechanical predictors *B* and *C* capable of measuring, with an appreciable degree of precision, all the individual particles of a mechanical system *A* (a gas, for example). But if this is so, then it is clear that a *mechanical* predictor *D* could escape 'detection' by *B* and *C* only if *D* is outside the reach of the instruments (tentacles) of *B* and *C*. But this is not possible; at least some mechanical parts of *D* must operate within the reach of *B* and *C*, and in attempting to predict the movements of these parts, *B* and *C* will strongly react to these movements. They may be completely unsuccessful, and this fact, for instance, may be 'known' to a sufficiently superior *D*, while only little about *D* may be 'known' to *B* or *C*. But since *D* cannot predict in detail the movements of its own parts, it cannot predict in detail the movements by which *B* and *C* react to the movements of its parts.

Of course, if *B*'s and *C*'s 'attention' are otherwise engaged than in attempting to predict *D*, or parts of *D*, then *D* may be more successful in predicting *B* and *C*.

These considerations assume classical particle mechanics—some kind of atomism without quanta. But even if we assume a continuity system of mechanics—with infinitely divisible material substances—even then would we obtain similar results. For in such a system we would still have to assume, for the parts (say, the measuring rods or tentacles) of a mechanical *D*, that there must be a lower limit to their thickness or mass, etc., since otherwise they would become too easily distorted. Or to put it in another way: in a mechanical world, no *D* can be at the same time effective and vanishingly small.

But even if one mechanical system *D* did exist which could escape 'detection' by *B* and *C*, this would not help. For this system would then constitute a non-predictable system from the point of view of mechanics. Only if we assume that *for every* mechanical system, there exists one which is not only superior to it but, at the same time, so subtle in its operations as to be undetectable, only then would the finite doctrine of determinism be valid.

But the infinite series of predictors in whose existence or physical constructability we would have to believe would be very different from the one described in section 4. It could not be convergent—

for in this case, all predictors coming after a certain predictor P_n would differ very little from P_n and would not therefore be sufficiently dissimilar to be undetectable by P_n. In other words, the series could not converge towards some ideal predictor P_∞, for otherwise we would have to assume that all the predictors of such a series from a certain P_n on belong to one 'society.' (Moreover, even the existence of P_∞ would not help.)

Our last considerations apply only to certain classical systems of mechanics, and are therefore of a less general character than our other arguments. They are intended to show that the attempt to rescue determinism leads to assumptions which, although not logically impossible, are not only highly implausible, but incompatible with certain classical systems of physics which means that classical mechanics is inconsistent with the finite determinist doctrine that for every specified prediction task, it is possible, from the point of view of the physical system under consideration, to construct an adequate predictor (see section 3). And they show that, if there are classical mechanical predictors (and otherwise predictions do not exist at all in a physical sense, from the point of view of classical mechanics), then there must exist unpredictable predictors—predictors which are so sensitive that they would be strongly disturbed by every attempt, of a classical mechanical predictor, to predict them.

I do not claim that an analogous theorem must hold for every other (classical) system of physics, although it appears to me very probable that this is indeed the case for every system which is sufficiently complete to contain a theory which can analyse the functioning of the predictor in question. One might think, for example, that in a classical system which includes a theory of light, one could construct, for every detector P_n of light signals, a sender P_{n+1} which sends signals of a wave-length which cannot be detected by P_n, so that P_{n+1} can observe P_n without P_n detecting this fact. But this, I believe, is connected with the fact that such a classical system of physics fails to give a satisfactory account of the interaction between matter and radiation, and therefore of permanent records which retain the results of optical measurements, such as photographic plates. This, it appears, can only be done by a non-classical theory such as quantum mechanics; but to a theory of this kind our argument appears to be applicable, for reasons analogous to those indicated above.

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(Part II will be published in the next number.)